Journal of Applied Mechanics and Technical Physics, Vol. 40, No. 1, 1999

NUMERICAL STUDY OF NONLINEAR DYNAMICS OF HYDROELASTICALLY CONNECTED, PLANE CURVILINEAR RODS

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UDC 539.3:621.039.58

The nonlinear dynamics of hydroelastically connected, plane curvilinear rods is studied. We take into account the reciprocal effect of deformation and hydrodynamics processes, large displacements and strains of the rods, the preliminary static stress-strain state, and the nonstationary fluid flow. A method is proposed for numerical solution of initial boundary-value problems. The effects of the hydroelastic interaction are investigated. The effect of various factors on the dynamics of a damaged pipeline is analyzed.

Extended fluid-filled pipelines are the structural elements of many energetic and industrial systems. The specific feature of the nonstationary behavior of these systems is the mutual influence of deformation and hydrodynamic processes. At the present time, the approach based on the equations of motion of hollow rods [1] is widely used to model the dynamics of pipelines. Moreover, the fluid is assumed to be incompressible and its inertia properties are taken into account to describe transverse and bending motions of the elastic rod [2]. The equations of motion obtained can describe large displacements of the axial line; however, they are not wave equations and their application to model, say, water-hammer processes, is problematic. At the same time, it is known that the wave equations [3] take into account the hydroelastic effects of deformation of rods and of an acoustically compressible ideal fluid more accurately. The equations are linear and are derived for the case of straight pipes of circular cross section.

In the present paper, we formulate a nonlinear system of dynamic equations of pipelines as curvilinear rods, which describe wave processes with allowance for large displacements and elastoplastic strains in pipes and the preliminary static stress-strain state. An algorithm for numerical solution of the initial boundary-value problems is developed and the effects of the hydroelastic interaction are studied.

Basic Equations. The axial line of the pipe (the curvilinear rod) is assumed to be a plane curve. We introduce two Cartesian coordinate systems (Fig. 1): the global fixed system rOz and the local moving system $\tau s \xi$ related to the deformable axis of the rod, where s is the length of the axial line from its origin to a current point ($0 \leqslant s \leqslant L$) and φ is the angle of rotation of the cross section. We assume the following expressions for the normal $\dot{u}^*_\xi$ and tangential $\dot{u}^*_\tau$ displacement rates over the rod thickness:

$$\dot{u}^*_\xi(s,\xi,t) = \dot{u}_\xi(s,t), \qquad \dot{u}^*_\tau(s,\xi,t) = \dot{u}_\tau(s,t) + \xi\dot{\varphi}(s,t),$$

where $\dot{u}_\xi(s,t)$ and $\dot{u}_\tau(s,t)$ are the displacement rates of the particles at the axial line and the dot over the symbol denotes differentiation with respect to time t. The axial $\dot{\varepsilon}$ and shear $\dot{\gamma}$ strain rates are expressed in terms of the displacement rates as follows:

$$\dot{\varepsilon}^0 = \dot{u}_{\tau,s}, \quad \dot{\varepsilon} = \dot{\varepsilon}^0 + \xi\dot{\varphi}_{,s}, \quad \dot{\gamma}^0 = \dot{\varphi} + \dot{u}_{\xi,s}, \quad \dot{\gamma} = \dot{\gamma}^0[1 - (\xi H^{-1})^2]. \tag{1}$$

Here $-H \leqslant \xi \leqslant H$ and $H = R + h$ (R is the inner radius of the pipe cross section and h is the wall thickness), and the subscript after the comma denotes the derivative with respect to the corresponding spatial variable.

Institute of Mechanics, Nizhnii Novgorod State University, Nizhnii Novgorod 603600. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 40, No. 1, pp. 212–219, January–February, 1999. Original article submitted January 20, 1997.

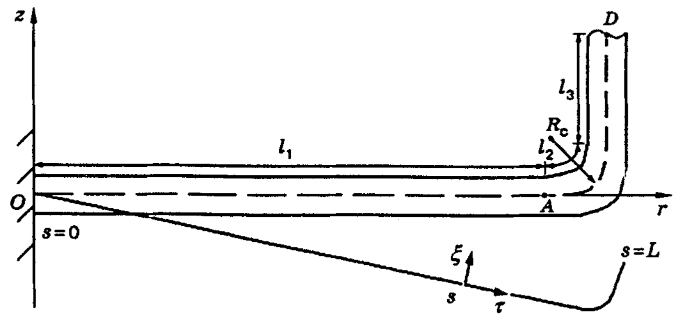

Fig. 1

The relation between the displacement rates in different coordinate systems is given by the formulas

$$\dot{u}_\tau = \dot{u}_z z_{,s} + \dot{u}_r r_{,s}, \qquad \dot{u}_\xi = \dot{u}_z r_{,s} - \dot{u}_r z_{,s},$$

where $z(s,t)$ and $r(s,t)$ are the coordinates of the axial line.

The elastoplastic deformation of the rod is described based on the equations of plastic flow with linear kinematic hardening:

$$\dot{\varepsilon} = \dot{\varepsilon}' + \dot{\varepsilon}'', \quad \dot{\gamma} = \dot{\gamma}' + \dot{\gamma}'', \quad \sigma_{11} = E\varepsilon', \quad \sigma_{13} = 0.5G\gamma', \quad \varepsilon = \int_0^t \dot{\varepsilon}\, dt, \quad \varepsilon'' = \int_0^t \dot{\varepsilon}''\, dt,$$

$$\gamma = \int_0^t \dot{\gamma}\, dt, \quad \gamma'' = \int_0^t \dot{\gamma}''\, dt, \quad S_{ij}S_{ij} = \frac{2}{3}\sigma^2_{\text{yield}}, \quad S_{11} = \frac{2}{3}\sigma_{11} - 2g\varepsilon'', \tag{2}$$

$$S_{13} = \sigma_{13} - g\gamma'', \quad S_{22} = S_{33} = -\frac{1}{3}\sigma_{11}, \quad \dot{\varepsilon}'' = \lambda S_{11}, \quad \dot{\gamma}'' = 2\lambda S_{13}.$$

Here the prime and double prime denote the elastic and plastic components of the tensors, respectively, E is the Young's modulus, G is the shear modulus, ν is the Poisson ratio, σ_{yield} is the yield point, g is the modulus of hardening of the structural material, and λ is a parameter which is equal to zero for elastic deformation and is determined for elastoplastic deformation from the condition that the instantaneous yield surface passes through the end of the after-loading vector in the space of the components of the stress deviator. The axial N and shear Q forces and the bending moment M are determined by integrating the corresponding stresses:

$$N = \int_{A_p} \sigma^*_{11}\, dA_p, \qquad Q = \int_{A_p} \sigma_{13}\, dA_p, \qquad M = \int_{A_p} \sigma_{11}\, dA_p. \tag{3}$$

Here A_p is the cross-sectional area of the rod, $\sigma^*_{11} = \sigma_{11} - \nu PRh^{-1}$ (the Poisson effect is taken into account [3]), and P is the pressure in the fluid flow. For thin-walled pipes, we set $A_p = \pi(R+H)h$. To derive the equations of motion of the pipeline, we use Gurtin's variational principle [4]:

$$\int_0^L \{(N\delta\dot{\varepsilon} + Q\delta\dot{\gamma} + M\delta\dot{\varphi}_{,s}) + \rho_p A_p(\ddot{u}_\tau\delta\dot{u}_\tau + \ddot{u}_\xi\delta\dot{u}_\xi) + \rho_p J_p\ddot{\varphi}\delta\dot{\varphi} + \rho_f A_f\ddot{u}_\xi\delta\dot{u}_\xi + \rho_f J_f\ddot{\varphi}\delta\dot{\varphi}\}\, ds$$

$$- \int_0^L (F_\tau\delta\dot{u}_\tau + F_\xi\delta\dot{u}_\xi + F^f_\xi\delta\dot{u}_\xi + F^f_\tau\delta\dot{u}_\tau)\, ds - [P^0_\tau\delta\dot{u}_\tau + P^0_\xi\delta\dot{u}_\xi + M^0\delta\dot{\varphi}]_{s=0,L} = 0. \tag{4}$$

Here ρ_p and ρ_f are the densities of the rod material and the fluid, respectively, J_p and J_f are the moments of inertia of the cross section of the rod $J_p = (1/8)\pi(R+H)^3h$ and the element of the fluid $J_f = (1/4)\pi R^4$ with respect to the plane which passes through the axial line and is perpendicular to the $\tau s\xi$ plane, A_f is the cross-sectional area of the fluid, F^f_τ and F^f_ξ are the components of the load per unit length exerted by the fluid flow on the rod, F_τ and F_ξ are the components of the external load per unit length, and P_τ^0, P_ξ^0, and M^0 are the boundary values of the forces and the moment. Substituting expressions (1) into (4), assuming that the variations $\delta\dot{u}_\tau$, $\delta\dot{u}_\xi$, and $\delta\dot{\varphi}$ are independent, and passing to the global fixed coordinate system, we obtain the system of differential equations

$$\rho_p A_p \ddot{u}_z - (N z_{,s} + k_m Q r_{,s})_{,s} = F_z + F_z^f, \quad \rho_p A_p \ddot{u}_r - (N r_{,s} - k_m Q z_{,s})_{,s} = F_r + F_r^f,$$
$$\rho_p J_p \ddot{\varphi} - k_J M_{,s} = -k_J Q \tag{5}$$

and the complete set of possible boundary conditions

$$\begin{array}{lll} M = \pm M^0 & \text{if} & \delta\dot{\varphi} \neq 0, \text{ or } \dot{\varphi} = \dot{\varphi}^0(t) \text{ if } \delta\dot{\varphi} = 0; \\ (N z_{,s} + k_m Q r_{,s}) = \pm P_z^0 & \text{if} & \delta\dot{u}_z \neq 0; \\ \dot{u}_z = \dot{u}_z^0(t) & \text{if} & \delta\dot{u}_z = 0; \\ (N r_{,s} - k_m Q z_{,s}) = \pm P_r^0 & \text{if} & \delta\dot{u}_r \neq 0; \\ \dot{u}_r = \dot{u}_r^0(t) & \text{if} & \delta\dot{u}_r = 0, \end{array}$$

where $F_r = F_\tau r_{,s} - k_m F_\xi z_{,s}$ and $F_z = F_\tau z_{,s} + k_m F_\xi r_{,s}$ are the components of the load per unit length in the fixed basis and $k_m = \rho_p A_p [\rho_p A_p + \rho_f A_f]^{-1}$ and $k_J = \rho_p J_p [\rho_p J_p + \rho_f J_f]^{-1}$ are the coefficients that take into account the mass and moment of inertia of the fluid. If $k_m = k_J = 1$, the inertia properties of the fluid are ignored. The initial conditions for system (5) are obtained from the solution of the problem of equilibrium of a curvilinear rod subjected to static loads, including the forces exerted by a steady fluid flow.

The nonstationary behavior of the fluid in the rod is assumed to depend on the coordinate s, the time t, and the longitudinal strains of the rod, and it is described by the modified acoustical equations

$$\dot{V} + \rho_f^{-1} P_{,s} = 0, \qquad \dot{P} + B V_{,s} - 2\nu B \dot{\varepsilon}^0 = 0 \tag{6}$$

with the initial conditions $P(s,0) = P_{\text{ini}}$ and $V(L,0) = V_{\text{ini}}$, where P and V are the pressure and velocity of the fluid flow, $B = \rho_f c_g^2$ is the bulk modulus of the fluid, and $c_g = c_f(1 + 2R\rho_f c_f^2 [hE]^{-1})^{-0.5}$ is the velocity of Joukowski's perturbations (c_f is the speed of sound in an infinite fluid). In deriving these equations, we assume that the pipe surface does not buckle during deformation and the cross section does not deform. As the boundary conditions for $s = 0$ and $s = L$, we specify the pressure and the velocity as functions of time.

The relation between the systems of dynamic equations (5) and fluid flow (6) is due to specifying the right-hand sides F_r^f and F_z^f in system (5) as functions of the current pressure and velocity of the fluid flow. The force per unit length which is exerted on the curvilinear sections of the rod ds by the fluid (the fluid friction on the pipe walls is ignored) has the following components in the local basis [2]:

$$F_\xi^f = -(\rho_f V^2 + P) A_f K \xi(s), \qquad F_\tau^f = 0. \tag{7}$$

Here $\xi(s)$ is the unit dimensionless vector normal to the axial line and $K = -r_{,s} z_{,ss} + z_{,s} r_{,ss}$ is the curvature of the axial line. Thus, the right-hand sides of system (5) can be presented as follows:

$$F_z^f = k_m F_\xi^f r_{,s}, \qquad F_r^f = -k_m F_\xi^f z_{,s}.$$

Equations (1)–(7) describe the hydroelastic wave deformation of a fluid-filled rod with allowance for large displacements of its axial line. The total displacements are determined by summation of the displacements obtained at each stage dt of the deformation process. Although system (5) looks like a linear system, it is in fact nonlinear, since the coordinates of the axial line $r(s,t)$ and $z(s,t)$ and, consequently, $r_{,s}$, $z_{,s}$, F_r^f, and F_z^f are the functionals of the deformation process. We note that this formulation of the problem takes into account the effect of spatial displacements of the pipe axial line on the wave processes in the fluid flow and vice versa.

Solution Algorithm. The problem posed is solved by numerical methods. The equations of motion (5) are integrated by the variational difference method using the explicit scheme "cross" [5]. The axial line is divided into N_s intervals of length $\Delta s = L N_s^{-1}$. A set of end points of the intervals, including boundary points, forms a basic grid. At these nodes, denoted by the integer index j, the coordinates, displacements, and displacement rates of the axial line are determined, while at the centers of the intervals, denoted by the ıalf index $j+1/2$, we determine the stresses, the strains, the forces, the moments, and their derivatives with espect to time. The subscript j and the superscript j refer to the quantities at the layers t_0 and $t = t_0 + \Delta t$, espectively. Replacing the variational equation (4) by the discrete analog and bearing in mind that the variations in the displacement rates at the nodes of the basic grid are independent, we obtain the explicit ecursive relations

$$\begin{aligned}
\dot{u}_z^j &= \dot{u}_{z_j} + \frac{\Delta t}{\rho_p A_p \Delta s}[(Nz_{,s} + k_m Qr_{,s})_{j+1/2} - (Nz_{,s} + k_m Qr_{,s})_{j-1/2}] \\
&\quad + \frac{\Delta t}{2\rho_p A_p}(F_{z_{j+1/2}} + F_{z_{j-1/2}} + F^f_{z_{j+1/2}} + F^f_{z_{j-1/2}}), \\
\dot{u}_r^j &= \dot{u}_{r_j} + \frac{\Delta t}{\rho_p A_p \Delta s}[(Nr_{,s} - k_m Qz_{,s})_{j+1/2} - (Nr_{,s} - k_m Qz_{,s})_{j-1/2}] \\
&\quad + \frac{\Delta t}{2\rho_p A_p}(F_{r_{j+1/2}} + F_{r_{j-1/2}} + F^f_{r_{j+1/2}} + F^f_{r_{j-1/2}}), \\
\dot{\varphi}^j &= \dot{\varphi}_j + \frac{\Delta t}{\rho_p J_p \Delta s} k_J [M_{j+1/2} - M_{j-1/2} - 0.5(Q_{j+1/2} + Q_{j-1/2})\Delta s], \\
\alpha^j &= \alpha_j + \dot{\alpha}^j \Delta t \qquad (\alpha = u_z, \quad \alpha = u_r, \quad \alpha = \varphi; \quad j = 1, \ldots, N_s).
\end{aligned} \tag{8}$$

In deriving the difference relations (8), the following approximations of the expressions for the strain rates of the axial line are used:

$$\begin{gathered}
(\dot{\varepsilon}^0)^{j+1/2} = [(\dot{u}_z^{j+1} - \dot{u}_z^j)(z^{j+1} - z^j) + (\dot{u}_r^{j+1} - \dot{u}_r^j)(r^{j+1} - r^j)](\Delta s)^{-2}, \\
(\dot{\gamma}^0)^{j+1/2} = 0.5(\dot{\varphi}^{j+1} + \dot{\varphi}^j) - [(\dot{u}_r^{j+1} - \dot{u}_r^j)(z^{j+1} - z^j) - (\dot{u}_z^{j+1} - \dot{u}_z^j)(r^{j+1} - r^j)](\Delta s)^{-2}, \\
\dot{\varphi}_{,s}^{j+1/2} = (\dot{\varphi}^{j+1} - \dot{\varphi}^j)(\Delta s)^{-1}.
\end{gathered}$$

For the elastoplastic problem, in addition to the grid along the axial line we introduce a grid in the transverse direction. The cross section of the pipeline is divided into N_ξ layers of equal thickness. The strains and stresses in the layers are determined by formulas (1) and (2). The plastic-strain components are calculated by iterations until the yield condition is satisfied. In this case, the forces and the moment (3) are calculated by Simpson's rule. We use the regularization [5] of the system of difference equations (8), which enables us to weaken the stability condition and provide better accuracy of numerical results for $\Delta s(R+H)^{-1} > 1$. Moreover, the time step is determined from the Courant condition for the longitudinal elastic waves:

$$\Delta t \leqslant \Delta t_K = \Delta s(E\rho_p^{-1})^{-0.5}. \tag{9}$$

The hydrodynamic equations (6) are integrated according to Godunov's explicit scheme [6]. The difference relations for the same grid along the axial coordinate have the form

$$\begin{gathered}
V^{j+1/2} = V_{j+1/2} - \frac{\Delta t}{\rho_f \Delta s}(P_{j+1} - P_j), \quad P^{j+1/2} = P_{j+1/2} - \frac{\Delta t}{\Delta s} B(V_{j+1} - V_j) - \Delta t 2\nu B(\dot{\varepsilon}^0)_{j+1/2}, \\
P_j = 0.5(P_{j+1/2} + P_{j-1/2}) - 0.5\rho_f c_g (V_{j+1/2} - V_{j-1/2}), \\
V_j = 0.5(V_{j+1/2} - V_{j-1/2}) - 0.5\frac{1}{\rho_f c_g}(P_{j+1/2} - P_{j-1/2}).
\end{gathered} \tag{10}$$

The stability condition for system (10) is weaker than that for (9) and, therefore, the coupled problem is solved with the step $\Delta t = \Delta t_K$. The interaction force exerted on the pipeline by the fluid flow (7) is approximated by the expression $F_{j+1/2} = -(\rho_f V_{j+1/2}^2 + P_{j+1/2})A_f K_{j+1/2}$, where the current curvature of the axial line $K_{j+1/2}$ is calculated by the formula

$$\begin{aligned}
K_{j+1/2} &= -0.5(r_{j+1} - r_j)(z_{j+2} - z_{j+1} - z_j + z_{j-1})(\Delta s)^{-3} \\
&\quad + 0.5(z_{j+1} - z_j)(r_{j+2} - r_{j+1} - r_j + r_{j-1})(\Delta s)^{-3}.
\end{aligned} \tag{11}$$

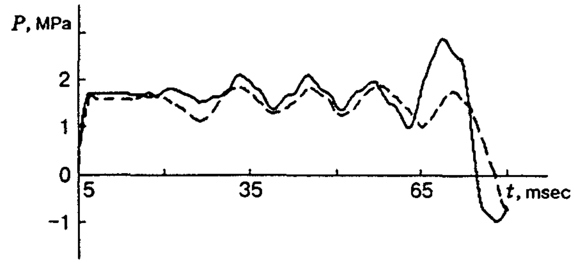

Fig. 2

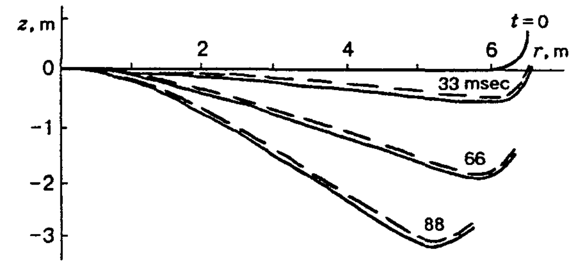

Fig. 3

Relation (11) is valid for the internal cells of the grid. The curvature on the boundary intervals is calculated using the corresponding truncated three-point schemes.

The preliminary static stress-strain state of the pipeline loaded with a steady fluid flow is determined by solving the dynamic problem by the establishment method [7] according to the above algorithm. The calculation procedure is iterative and consists of the following. In the first iteration, the unstressed rod starts moving and deforms under the action of stationary forces. When the total kinetic energy of the rod

$$W_i = 0.5\rho_p L \sum_{j=0}^{n} \{A_p[(\dot{u}_r)_j^2 + (\dot{u}_z)_j^2] + J_p \dot{\varphi}_j^2\} \qquad (i = 1, \ldots, m)$$

reaches the global maximum, the displacement rates vanish, $(\dot{u}_z)_j = (\dot{u}_r)_j = (\dot{\varphi})_j = 0$, and the transition to the next iteration occurs, in which the stress-strain state of the rod obtained in the preceding iteration is taken to be the initial condition; then the dynamic problem is solved again until the kinetic energy W_{i+1} attains the maximum value. The process of finding an equilibrium state is repeated until the condition $W_{i+1}(W_1)^{-1} \leqslant \delta$ is satisfied, where W_1 and W_{i+1} are the maximum values of the total kinetic energy for the first and $(i+1)$th loading cycles and $\delta = 10^{-5}$. The resulting static stress-strain state is used to solve the problem of nonstationary deformation of the pipeline.

Verification of the Numerical Algorithm. To test the developed algorithm we compare the numerical results of [3], where the water-hammer problem in a pipeline was solved for the case of a quick closure of the valve at one end. The parameters of the pipeline (Fig. 1) are as follows: $l_1 = 35.6$ m, $l_2 = 0.31$ m, $l_3 = 12.3$ m, $R_c = 0.2$ m, $R = 1.25$ cm, $h = 0.127$ cm, $E = 1.17 \cdot 10^5$ MPa, and $\rho = 8940$ kg/m^3. For $s = 0$ and $s = L$, the fixed-end conditions $\dot{u}_z = \dot{u}_r = \dot{\varphi} = 0$ are specified. The fluid flow is assumed to be nonstationary with $c_f = 1500$ m/sec, $\rho_f = 1000$ kg/m^3, and the initial and boundary conditions $P_{\rm ini} = 0.4$ MPa, $V_{\rm ini} = 1$ m/sec, $P(0,t) = 0.4$ MPa, and $V(L,t) = \{V_0(1 - t/T)$ for $t \leqslant T$; 0 for $t > T\}$ modeling the constant fluid replenishment at the end $s = 0$ from a constant-pressure reservoir and the closure of the valve D at the time $T = 0.2$ msec at the end $s = L$. At the initial moment of time, the displacements, stresses, and strains do not occur in the pipeline. The dependence of the fluid pressure on time in the vicinity of the valve is shown in Fig. 2. The solid curve refers to the algorithm in question, and the dashed curve to the results of [3]. On the whole, satisfactory agreement between the results is observed, which confirms the reliability of the numerical solutions. For these parameters of the problem, the pipe deforms elastically. The discrepancy between the results can be explained by the incomplete definiteness of the problem formulation in [3] and the different computational algorithms used.

Figure 3 shows the results of analysis of the dynamics of the pipeline of an atomic power station when there is a break in its total cross section with the use of the above algorithm (solid curves) and the model of [8] (dashed curves). The pipeline parameters are as follows (see Fig. 1): $l_1 = 5.84$ m, $l_2 = 0.864$ m, $l_3 = 0.175$ m, $R_c = 0.55$ m, $R = 14.56$ cm, $h = 1.6$ cm, $E = 2 \cdot 10^5$ MPa, $\nu = 0.3$, $\rho = 7800$ kg/m^3, $\sigma_{\rm yield} = 400$ MPa, and $3g = 2 \cdot 10^3$ MPa. To model the break, we set the fixed-end boundary conditions $\dot{u}_z = \dot{u}_r = \dot{\varphi} = 0$ for $s = 0$ and the free-end boundary conditions $P_z^0 = P_r^0 = M^0 = 0$ for $s = L$. For $t = 0$, the pipeline is stress-free. The fluid flow is assumed to be steady with $c_f = 1500$ m/sec, $\rho_f = 1000$ kg/m^3, $P_{\rm ini} = 5.05$ MPa, and $V_1 = 7.92$ m/sec.

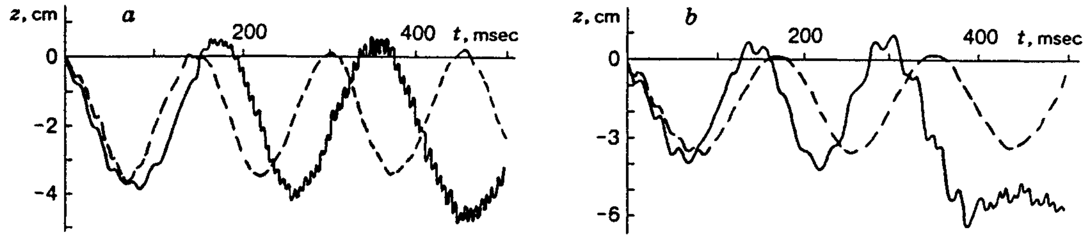

Fig. 4

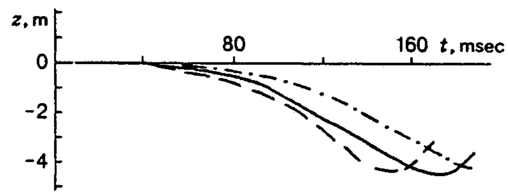

Fig. 5

If the pipeline undergoes large displacements, plastic deformation occurs in the neighborhood of its fixed end. The configurations of the pipeline at the moments $t = 0$, 33, 66, and 88 msec are shown. The results obtained by the above algorithm and by the model of [8] agree well, which supports the reliability of the numerical solutions obtained in this simplified formulation.

Analysis of Hydroelastic Effects. Figures 4 and 5 show the calculation results obtained for refined formulations of the problem. The time variation of the coordinate z of the point A is shown. Figure 4a shows the numerical solutions for the complete hydroelastic (solid curve) and uncoupled (dashed curve) formulations. In both cases, the fluid flow is assumed to be nonstationary with the initial data $P_i = 5.05$ MPa and $V_i = 7.92$ m/sec and the boundary conditions $V(0,t) = 7.92$ m/sec for $s = 0$ (constant flow rate) and $P(L,t) = 0.1$ for $s = L$ (free outflow); the preliminary stress-strain state is omitted. In the uncoupled formulation, we set $k_m = k_J = 1$ and $\nu = 0$ in the hydrodynamic equations (6) and in determining the axial force (3). A considerable discrepancy is seen between the amplitudes and phases of the vibrations of the damaged pipeline. A more complete solution involves high-frequency harmonics related to the axial vibrations of the pipe due to the Poisson effect. In hydroelastically coupled vibrations, the amplitudes tend to increase, which results in the occurrence of plastic deformation in the neighborhood of the fixed end of the rod. To analyze the influence of the hydroelastic effects in greater detail, the problem was calculated in simplified but coupled formulations. The solid curve in Fig. 4b refers to the solution with $k_m = k_J = 1$ but $\nu \neq 0$, and the dashed curve to the solution with $\nu = 0$ but $k_m \neq 1$ and $k_J \neq 1$. It is seen that allowance for the properties of inertia of the fluid significantly alters the period of vibrations of the system; however, the stresses in the rod do not exceed the yield point. Meanwhile, the allowance for the Poisson effect increases the amplitude of vibrations and results in the occurrence of residual deformations and a qualitative alteration of the character of motion of the system.

To analyze the influence of the hydroelastic effects at large displacements of the pipeline, we considered a different problem: a different boundary condition was imposed on the hydrodynamic equation for $s = 0$, namely, $P(0,t) = 5.05$ MPa, i.e., the left end of the pipeline was assumed to be connected to a large reservoir of constant pressure. In this case, the hydrodynamic load acting on the damaged pipeline increased significantly. Figure 5 shows the predicted displacement of the point A, where the solid curve corresponds to the complete formulation of the hydroelastically coupled problem, the dashed curves to the uncoupled problem, and the dash-and-dotted curve to the solution of the coupled problem with account of the preliminary stress-strain state. With allowance for the properties of inertia of the fluid and $\nu = 0$, the solution of the problem almost coincides with that of the complete coupled problem. Thus, in the case of large displacements of the damaged pipeline the inclusion of the inertia of the fluid inside the pipe is of greater importance. It is seen that when the preliminary stress-strain state is taken into account, the initial stresses and strains are the stabilizing factors which retard the development of the processes, although the subsequent failure of the damaged pipeline is also obvious in this case.

The above-described mathematical and numerical models make it possible to analyze damaged highly pressurized pipelines with allowance for large displacements and strains, the interconnection of deformation and hydrodynamic processes, and the preliminary static stress-strain state. These factors should be taken into account to model and predict emergencies at highly pressurized pipelines.

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